The use of ludic resources for the teaching of probability and statistics in middle school

A utilização de recursos lúdicos para o ensino de probabilidade e estatística no ensino médio

El uso de recursos lúdicos para enseñar probabilidad y estadística en la escuela secundaria

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Abstract

Bearing in mind that the teaching of statistics and probability enables critical development in students, this experience article presents the handling of playful resources aimed at the mathematics classroom, whose target audience was high school students, in which the objective was to enhance students' perception of statistics present in their daily lives. Regarding the methodology, studies related to critical statistical education were used as a theoretical contribution in which they defend the teaching of statistics related to problematization and critical awakening as an instrument in development when observing these concepts present in everyday life, since it is knowledge that the teaching of statistics through the mathematics teacher is often not presented or demonstrated without intertwining it with situations experienced daily. The students' report showed that the use of playfulness to employ the concept of statistics and probability in conjunction with problematization and the possibility of group discussion favored significant learning.
**Keywords:** Critical education; Statistical education; Mathematical education; Teaching; Recreational resources.

**Resumo**
Tendo em vista que o ensino de estatística e probabilidade viabiliza o desenvolvimento crítico nos educandos, este artigo de experiência apresenta o manuseio de recursos lúdicos voltados para a sala de aula de matemática, cujo público alvo foram alunos do ensino médio, em que o objetivo foi de potencializar nos alunos a percepção da estatística presente no dia-a-dia. Em relação à metodologia, os estudos relacionados à educação estatística crítica foram utilizados como aporte teórico em que estes defendem o ensino de estatística ligado a problematização e o despertar crítico como uma instrumentalização no desenvolvimento ao observarem estes conceitos presentes no cotidiano, uma vez que é de conhecimento que o ensino de estatística por meio do professor de matemática muitas vezes não se apresenta ou demonstram estes sem o interlaçar com situações vivenciadas diariamente. O relato dos alunos evidenciou que a utilização do lúdico para empregar o conceito de estatística e probabilidade em conjunto com a problematização e a possibilidade da discussão em grupo favoreceu a aprendizagem significativa.

**Palavras-chave:** Educação crítica; Educação estatística; Educação matemática; Ensino; Recursos lúdicos.

**Resumen**
Teniendo en cuenta que la enseñanza de la estadística y la probabilidad posibilita un desarrollo crítico en los estudiantes, este artículo de experiencia presenta el manejo de recursos lúdicos dirigidos al aula de matemáticas, cuyo público objetivo fueron los estudiantes de secundaria, en el que el objetivo para mejorar la percepción de los estudiantes de las estadísticas presentes en su vida diaria. En cuanto a la metodología, se utilizaron como aporte teórico estudios relacionados con la educación estadística crítica en los que defienden la enseñanza de la estadística relacionada con la problematización y el despertar crítico como instrumento en desarrollo al observar estos conceptos presentes en la vida cotidiana, ya que es conocimiento de que la enseñanza de la estadística a través del profesor de matemáticas a menudo no se presenta o demuestra sin entrelazarla con situaciones vividas a diario. El informe de los estudiantes mostró que el uso de la alegría para emplear el concepto de estadística y probabilidad junto con la problematización y la posibilidad de discusión en grupo favoreció el aprendizaje significativo.
Palabras clave: Educación crítica; Estadísticas de educación; Educación Matemática; Enseñanza; Recursos lúdicos.

1. Introduction

Currently, statistical education has been a field of research highly explored by educators, since the teaching of statistics is fragmented in basic education, which is often overlooked by mathematics teachers due to several situations, such as: teacher training, the lack of methodological resources and the range of contents to be fulfilled in the school year (Lopes, 2008).

Regarding the direction of the mathematics teacher in the classroom, according to the National Curriculum Parameters (PCN) (Brasil, 1998), the content of statistics is located in the section called “Information Treatment”, in which the importance of this section is seen, which in turn instead, if explored in classrooms, it enables and enhances the formation of critical citizens, since the concepts included in this block refer to data reading, interpretation, analysis of situations and decision making.

As highlighted by Wodewotzki et al. (2010), Walichinski (2013) and Borges et al. (2019), the teaching of statistics helps students to plan data collection, to interpret and analyze the data obtained and to present the results in order to assist them in making decisions. Lemos (2011) and Yumi Kataoka et al. (2011) state that teaching statistics is not just numbers and graphs, but a way to awaken in students the practice of questioning/why, thus allowing a clear and objective description of nature's phenomena.

Castro & Cazorla (2007) and De Oliveira Mendonça (2011) also emphasize that statistical knowledge helps in the economic understanding related to the market, since large companies choose the most attractive statistics, tables and graphs to convince consumers to choose their cause, your good or service. In this sense, Lopes (2011) considers that Statistical Education contributes to an individual being able to analyze and critically relate the data presented, using statistical and probabilistic knowledge, in order to question and even verify the veracity of certain data.

Castro & Cazorla (2007) point out that the training of students regarding statistical education is not presented in a satisfactory and intensive way as seen in the academic environment. The authors further emphasize that:
When the speeches, advertisements, headlines and news carried by the media, use statistical information (numbers, tables or graphs), these gain credibility and are difficult to be challenged by ordinary people, who even question the veracity of this information, but not is equipped to argue and against argue (Castro & Cazorla, 2007).

Although in the current scenario there is a development/advance in the scientific environment in relation to the production of materials to be used in mathematics classrooms in order to help the mathematics teacher to employ statistical and probabilistic concepts, however there is an inherent difficulty in inserting them.

The studies by Campos (2011), Santana (2011) and Borba et. al (2011), show that a way to explore the concept previously mentioned is through the critical awakening of the student, in which the aforementioned author calls: Theory of Critical Statistical Education. Still according to Campos (2011), the concepts of statistics must be initiated by the mathematics teacher through a problematization, because just as the mathematical concepts must be associated with the students’ daily lives, statisticians must also do so. The methodological proposal that supports this article is based on the studies by Skovsmose (2006), Lopes (2011) and Campos (2011), in which these authors defend the teaching practice linked to critical education. Skovsmose (2006) advocates that:

Ideas related to dialogue and the student-teacher relationship are developed from the general point of view that education must be part of a democratization process (Skovsmose, 2006).

Thus, a teaching that values Critical Mathematical Education must be associated with instruments that can be used by students with the objective of helping them, both in the analysis of a critical situation as well as to solve certain situations. In this sense, not only should students be taught to use mathematical concepts, but rather, they should be asked to question why, how, for what and when to use them. Given what has been described, teaching is seen as a practice of developing critical awakening in the student.

According to Skovsmose (2006), Critical Education proposes some stages of development linked to the critical curriculum, which outline the investigation: the applicability of the subject, the interests behind the subject, the assumptions behind the subject, the functions of the subject and the subject limitations. For Skovsmose (2007), the teaching and learning process needs to be focused on problem solving. Such problems must prove to be important to students, be accessible to their previous knowledge and related to existing social problems.
The PCNs state that the contents of the Information Treatment block help the development of particular thoughts and reasoning and that they pay attention to the development of reasoning related to statistical and probabilistic thinking (Brasil, 1998), and more,

[…] reading and interpreting graphs, students get used to observing some aspects that allow them to trust or not the results presented […]. Data manipulation is usually frequent in statistical summaries, which are presented in inadequate graphs, which leads to errors of judgment. These errors can be avoided, if students are used to identifying the information that was raised, as well as complementary information, to prove mistakes that are made when collecting data, to check information to reach a conclusion (Brasil, 1998).

However, they do not seek to emphasize any of these competencies. Therefore, it is of utmost importance that discussions take place regarding statistical and probabilistic reasoning, taking into account that it is one of the main objectives of Statistical Education to develop them. In this context, this experience report aims to present the discussions regarding an application involving statistical concepts through playful resources, whose purpose of this application was to use mathematical investigation as a tool in the discovery and construction of statistical and probabilistic concepts, in a state school located in the city of Maringá – PR: Brazil, in the area of educational research, thus performing the intersection with the objectives of Statistical Education.

2. Literature Review

In order to obtain a broader look at the research on the subject, we conducted a survey at the Brazilian Digital Library of Theses and Dissertations (BDTD). Our search was carried out on October 20, 2020, and the keywords searched were “statistical education” “teaching statistics”, which resulted in 36 jobs. Highlighting the importance of setting a clear objective about the research in order not to deviate from it, Cervo, Bervian & Silva (2007) mention that informative reading is defined as an essential stage of collecting information on the topic to be investigated, which classify some steps for informative reading, which are: pre-reading, selective reading, critical or reflective reading, interpretive reading, and text comments.

Given what was previously described, we performed the pre-reading and the selective reading simultaneously, by reading the abstracts of the 46 works. In this initial stage, we selected five papers, all dissertations. Then, we carry out critical reading, interpretive and
comments simultaneously, discussing these briefly, as they have similar characteristics, in terms of the use of playful resources, media or quasi-experiments in the teaching of statistics.

Pagan (2010) aimed at comparing the learning gains of three groups of students in the 1st grade of High School, through a quasi-experiment, in which they had contact with elementary concepts of statistics. To this end, the author applied two diagnostic tests (pre and post-test) followed by a single teaching intervention, in which her methodology was based on the theory of Semiotic Representation Records. The results showed that the teaching of Statistics, linked to interdisciplinary concepts, brought significant answers as to the students' interest in learning about this theme.

Silva (2015) aimed to present a reflection on the pedagogical practices of the mathematics teacher in the teaching of statistics in high school, based on the guidelines of the Curriculum Parameters of High School, on the potential of significant learning in the teaching of statistics through technologies involving work plans that addressed the content of statistics and the use of free software Calc, in which the methodology was based on the deductive approach method. With the results obtained, the author concludes that it was possible to identify methodological paths to be developed by mathematics teachers in basic education involving the interpretation of data through tables, graph analysis, descriptive measures using Information and Communication Technologies as tools.

Morais (2017) used digital media in his dissertation to identify the perception of 7th grade students of a public school in the city of Cambé-PR: Brazil with regard to the content studied in statistics classes. To this end, the methodology adopted was of a quantitative/qualitative descriptive nature and included bibliographic research and field research carried out within the school, involving applied questionnaires, recordings and notes by the students. The digital resources inserted in the expository classes were video lessons prepared specifically with each content and inserted in a statistics vlog. With the results obtained, the author concludes that, through the instruments already described, improvements in learning and understanding of the contents of some statistical points were identified.

In order to experience Mathematical Modeling as a learning environment, Machado (2017) carried out a qualitative research involving didactic sequences, involving statistical concepts associated with day-to-day activities, based on questions directed to the reflection and investigation of students applied in a 7th grade class of elementary school in a public school in Sapucaia do Sul-RS: Brazil. The author can perceive during the elaboration and application of the didactic sequences that there was an evolution in the understanding of the
contents covered, which is justified in the Learning Environments provided by Mathematical Modeling.

This being the most recent work of this selection, Oliveira (2019) investigated “[...] the contributions of the poker game to facilitate the learning of the content of combinatorics and probability in high school”. The author points out that it is necessary to be careful on the part of the teacher so that the focus of learning is not lost amid playfulness, but that when used properly, the game enhances learning and increases the involvement of the class even in the expository classes.

Therefore, through these readings, the relevance of using educational tools is emphasized in order to stimulate students to understand and realize the importance of teaching statistics in daily life, in which through them the development of critical thinking about information that can be interpreted on a daily basis.

3. Methodology

3.1 Potentialities of teaching statistics in basic education

Regarding the scientific production involving statistical education, in the last years there has been an advance in this area aiming to present methodologies for working with statistics in basic education and its potential inside and outside the classroom, indicating that the teaching of statistics has been developing as an emerging theme (Garfield & Ben-Zvi, 2008). However, Zieffler et al. (2008) emphasize that conducting research involving the theme already mentioned involves a series of limiting factors for both the teacher and the student, highlighting the lack of teacher training and the need for basic math concepts by students.

With the expanding technological era and the generation of data as a consequence, it is noted that statistical literacy becomes an essential competence, which (Watson, 1997) mentions that the ability to process statistical information is necessary to understand the facts that occurred in the modern society, that is, such competence helps citizens to gather information and make daily decisions based on the information derived from this data, highlighting the media coverage, financial decisions, among others (Garfield & Ben-Zvi, 2007).

In the literature, it can be seen that a significant number of educators have dedicated themselves to understanding the challenges of learning and teaching statistics in mathematics
classes in order to identify the potential that this presents for the development of the student with regard to logical reasoning and obstacles faced on a daily basis regarding the interpretation of a table in a market, the understanding of an electoral result in a newspaper, as can be seen in Pagan (2010), Silva (2015), Morais (2017), Machado (2017), Oliveira (2019) among others.

Therefore, as already described and it is seen that in the current scenario the teaching of statistics has become an area of research that arouses interest on the part of educators, it is of paramount importance to implement new teaching tools in classrooms with the aim of achieving a balance between theory and application. Thus, in this experience report, we will present the results obtained through a workshop given at a public school in a city located in the interior of the state of Paraná involving concepts of statistics addressed in the classroom by mathematics teachers using playful materials.

3.2 Developed activities

Fifteen students from the second year of high school participated in the workshop (diagnostic study), from a public school in the city of Maringá, in the State of Paraná - Brazil. The material used was previously prepared and from them, three activities were carried out, namely:

1. Construction of bar graphs;
2. Construction of a scatter plot;

3.2.1 Construction of bar graphs

For the first activity, several cubes of different colors were arranged and each participant was asked to select two cubes of the color of their preference, both pieces having to be of the same color. After the selection the pieces were stacked according to the colors, forming the columns of the bar graph. Then, questions about the conformation of the bars were raised, these being: the highest frequency observed, if we stack the information what would happen with the interpretation, emphasizing that this graph is used to make comparisons between the categories of a discrete qualitative or quantitative variable, as well as the interpretation of this results.
The next proposal was to check how the graphic configuration would look when deciding to classify the selected colors by gender. From the observations, the participants were encouraged to relate the conclusions of the results obtained with other examples, in addition to what was commented, in which the representation through a bar graph could be used.

**Figure 1.** Construction of bar graphs.

3.2.2 Construction of a scatter plot

The second activity consisted of creating a scatter plot on a styrofoam board using pins for the mural. This activity was carried out by dividing them into three small groups. Each group received a set of information described below:

Group 1:

The “melanoma” data set contains information about 205 patients in Denmark with malignant melanoma. A subset of 30 observations was removed from this sample to verify whether age, in years, influences the patient's survival time, measured in days. The data can be seen in Table 1:
Table 1. Survival time of melanoma patients.

<table>
<thead>
<tr>
<th>Time</th>
<th>Age</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>76</td>
</tr>
<tr>
<td>30</td>
<td>56</td>
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<tr>
<td>35</td>
<td>41</td>
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<td>.</td>
<td>.</td>
</tr>
<tr>
<td>869</td>
<td>42</td>
</tr>
</tbody>
</table>

Source: Odense University Hospital, Denmark (1977)

Group 2:

The cabbage data set has 60 observations and 4 variables. A subset of 30 observations was taken from this sample to verify whether the size of the cabbage has an influence on the amount of vitamin C. The variable “Weight” is the weight of the cabbage head, presumably measured in kg; “Vit”, vitamin C content in undefined units; “Cult” refers to the type of cabbage grown, c39 and c52 and “Date” specifies the planting date, d16 or d21. The data can be seen in Table 2:

Table 2. Studies with data on cabbage production

<table>
<thead>
<tr>
<th>Cult</th>
<th>Date</th>
<th>Vit C</th>
<th>Weight</th>
</tr>
</thead>
<tbody>
<tr>
<td>c39</td>
<td>d16</td>
<td>2.5</td>
<td>51</td>
</tr>
<tr>
<td>c39</td>
<td>d16</td>
<td>2.2</td>
<td>55</td>
</tr>
<tr>
<td>c39</td>
<td>d16</td>
<td>3.1</td>
<td>45</td>
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<td>.</td>
<td>.</td>
</tr>
<tr>
<td>c52</td>
<td>d21</td>
<td>1.6</td>
<td>72</td>
</tr>
</tbody>
</table>


Group 3:

A study was carried out with 2287 eighth grade students (aged 11 years) in 132 classes from 131 schools in the Netherlands regarding verbal IQ, given how the ability to communicate and express themselves properly and the score in one language test. A subset of 30 observations was removed to verify whether verbal knowledge influences the IQ test score. The data can be seen in Table 3:
Table 3. Verbal IQ and language test.

<table>
<thead>
<tr>
<th>QI</th>
<th>Pointing</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>76</td>
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<tr>
<td>30</td>
<td>56</td>
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<tr>
<td>35</td>
<td>41</td>
</tr>
<tr>
<td>869</td>
<td>42</td>
</tr>
</tbody>
</table>

Source: Snijders e Bosker, Holanda, (1999)

In order to observe the behavior of variables with positive, negative and non-correlated correlations, the data sets pointed to different correlations. The choice of which observations would be the variables represented by the abscissa and ordered axes was decided by each group, emphasizing that they should justify the motivation of such choices. Finishing the activity, it was proposed to evaluate the configuration of the graphs when deciding to change the order of the ordered pair \((x, y)\) and if the conclusions would be the same. The development can be seen in Figure 2.

Figure 2. Construction of a scatter plot.

Source: Authors.

3.2.3 Resolution of probability problems

For the development of this activity, each group received a set of pieces that formed a circle in pink, green and blue. These parts would serve as aids in solving the problems below:

Problem 1: In a celebration, a pizza of three different flavors was served, here called "A", "B" and "C" flavors. If a person can serve himself freely, a) what is the probability that the first selected slice is of flavor A?
b) what is the probability that the first slice selected is of flavor B?
c) what is the probability that the first selected slice will taste C?

Problem 2: In a celebration, a pizza of three different flavors was served, here called "A", "B" and "C" flavors. If a person can serve freely and independently of the guest, what is the probability of:

a) the first two slices selected are of flavor A (B, C)?
b) the first three selected slices are of flavor A (B, C)?
c) the first two slices selected are of the same flavor?

The number of pieces was the same for all groups, however, the division by colors was different, so they would find different probabilities for the same event in each group. Problem 1 is solved by calculating simple probability while Problem 2 is solved by calculating conditional probability. Figure 3 illustrates the dynamics of the activity.

Figure 3. Construction of pie charts.

Source: Authors.

4. Results and Discussion

The first activity was the construction of a simple bar graph, where the information to be represented there was the color preference of those present. In order to construct the ideas involved in the activity, regardless of the mathematical concept on which they are based, the formal concept of bar graph was not defined at first. Asked about what they could get from information about what was represented there, the participants managed to reach pertinent conclusions, such as “the favorite color of the group is pink” and associate it with the reality
of that moment: “there are more girls in the group than boys, and pink is usually the favorite color for girls”.

Based on this last conclusion, they were asked to redo the graph, but now separating the batteries into two groups: boys and girls. By separating in this way, they could observe that the previous inferences were pertinent and, in addition, different situations were raised by the class, relating the daily life to the activity performed. Information disseminated by means of communication, such as newspapers, internet and magazines, about various themes, were also associated with the graphic resource as a means of visual representation.

For the construction of the dispersion graph, there was a longer time of dedication, as well as an extensive discussion to decide which would be the dependent and the independent variable. The group that received the data set on the survival time of patients with melanoma encountered some difficulties, both in the interpretation of the text and in deciding which variable would influence the other, even so, the time of discussion until the dispersion graph was finalized, was lower than in the other groups. The arrangement of data in the graph indicated that there was no trend, so they could conclude that the survival time of patients with melanoma did not depend on the patient's age.

The groups that received the data set on crab morphological measurements and studies with cabbage data demonstrated the difficulties already mentioned, but which proved to be quite productive. With regard to cabbage measures, the main question was whether the vitamin C content influences the size of the cabbage or vice versa. The graph was constructed observing the statement of the problem: “check if the size of the cabbage influences the amount of vitamin C”, that is, x being the variable size of the cabbage and y, the vitamin C content, but they observed that, depending on the the researcher's interest, the axes could be changed. The graphical configuration showed a negative correlation, meaning that the larger the head of cabbage, the lower the vitamin C content. Even after completing the construction of the graph, they pointed out that they could analyze the relationship of the other variables in the data set relating to the weight variable and/or vitamin C.

The third group needed the help of a facilitator to interpret the problem and, like the second, it verified how the relationship between the variables occurred, questioning whether the verbal IQ score depended on the language test score or the other way around. After a more detailed definition by the facilitator, the graph could be finalized, resulting in a positive correlation: the more articulated with regard to verbal IQ, the better the student's language test score.
The results reached by the groups were shared in the larger group. Observations were added relating the results to some previous assumptions, such as, for example, believing that the life span of cancer patients would be shortened in older patients or that the larger the cabbage, the higher the vitamin C content. Other more common situations that could be observed by a scatter plot were cited, both by the facilitators and by the participants, finalizing the conclusions on the theme.

The resolution of probability problems, also performed in small groups, did not present major problems, at first, being easily solved when it came to simple probability. With regard to conditional probability, the groups needed to be monitored more closely, as the topic was new among them. The use of parts to illustrate the solution to the problem proved to be quite efficient. Although there were only three questions from each topic, the time until the problem was solved was reasonable, partly due to the time that the facilitators paused to incite reasoning among the participants, quite successfully, judging by what the students questioned and reproduced. The solutions were discussed with everyone and the reason why the results between the groups were different was observed.

4.1 Narrative

At the end of the workshop, an evaluation was carried out to verify the impressions caused to the participants. According to the evaluations, all students stated that the activities performed made the content more understandable and that the manipulation of objects collaborated to keep them interested. The activity that received the most positive comments was the one that involved calculations of probabilities, which surprised the creators of the workshop, as it is not uncommon for students to be resistant to this topic. Among the reports of the evaluations, we rewrote two:

“I liked making the bar graph better, because I didn't know how to interpret it”.
“The activity I liked most was probability. It became easier to understand using the pieces”.

5. Final Considerations

The teaching of statistics is seen as an important tool in today's society, we believe that the insertion of its concepts and procedures contributes to the formation of the citizen,
especially the student of basic education who will soon be inserted in the labor market, social and cultural interaction and policies. In order to exercise citizenship, especially in a society that carries a lot of information, it is essential that students know how to communicate / expose ideas, build and interpret tables and graphs, make logical inferences and estimates and analyze data and information. We also confirm that statistics have a significant importance in the development of these skills, so we chose to deepen and reflect on the topic through an investigation.

The initial procedure of this workshop was to build knowledge about the themes developed through the students themselves, in this way, the participants were encouraged to arrive at the results through questioning and observation of resources, without the concern of conceptualizing mathematical definitions in a rigid way. It was observed that the simple manipulation of the pieces allowed, in addition to the assimilation of the content, interaction between participants and facilitators and critical observations regarding the issues to be solved, relating, including to the social context to which they are inserted.

Although in this study the observational unit focused on high school students, such tools can also be applied in elementary school I and II, taking into account the students' conceptual limitations until then. Therefore, research in this scenario can be carried out, in which steps must be followed, which are: application of an initial questionnaire to ascertain the students' prior knowledge, if the objective is to use the playful materials to establish the concept. In cases to introduce the content, it is necessary to carry out a dynamic centered on establishing paths for the construction of knowledge, whether through workshops, problem solving, modeling in mathematics education, among others and more, such approaches may also expand on other concepts contained in the curriculum of students in elementary school, such as: measures of central tendency, multiplicative principle and simple arrangement.

It is important to note that the more they were able to manipulate the objects, the greater the interest and participation in the activities. It was not noticeable, in general, bored or discouraged during the workshop, which reinforces the opinion that playful resources are of great value in the teaching-learning process.

References


**Percentage contribution of each author in the manuscript**

Breno Gabriel da Silva – 33,33%
Yana Miranda Borges – 33,33%
Rafaela Galo – 33,33%