Influence of moisture content on the dynamic modulus of elasticity
Influência do teor de umidade no módulo de elasticidade dinâmico
Influencia del contenido de humedad en el módulo dinámico de elasticidad

Abstract
Pinus forests are the second most planted in Brazil and the wood obtained from them is mainly used as structural elements in the civil construction sector. The Brazilian standard that deals with calculations of wooden structures projects recommends the use of wood in the range of 10% to 20% of moisture content. Thus, this study aimed to present an investigation about the influence of moisture content on the static modulus of elasticity (MOE\textsubscript{st}) of wooden beams by transverse vibration tests. The MOE\textsubscript{st} was evaluated by static bending tests and transverse vibration tests for various moisture content. The transverse vibration test was performed in a free condition using an accelerometer and an impulse hammer, and the dynamic modulus of elasticity (MOE\textsubscript{d}) was determined from the frequency of the first bending vibration mode. A total of 20 wooden beams were obtained from freshly cut Pinus spp. logs with initial high moisture content were used, with nominal dimensions of 5 cm × 10 cm × 200 cm (thickness × width × length). It was observed that the MOE\textsubscript{d} is also affected by moisture content according to the non-linear regression model. There were significant changes in the dynamic modulus of elasticity (MOE\textsubscript{d}) for the moisture contents (M) below 25% (near the fiber saturation point - FSP).

Keywords: Bending test; Non-destructive test; Pinus spp. wood; Frequency; Fiber saturation.

Resumo
As florestas de pinus são as segundas mais plantadas no Brasil e sua madeira é utilizada principalmente como elementos estruturais no setor da construção civil. A norma brasileira que trata de cálculos de projetos de estruturas de madeira recomenda o uso de madeira na faixa de 10% a 20% de umidade.essa forma, foi objetivo do trabalho apresentar uma investigação sobre a influência do teor de umidade no módulo de elasticidade estático (MOE\textsubscript{st}) de vigas de madeira por meio de ensaios de vibração transversal. O MOE\textsubscript{st} foi avaliado por ensaios estáticos de flexão e ensaios de vibração transversal para diversos teores de umidade. O ensaio de vibração transversal foi realizado em condição livre utilizando um acelerômetro e um martelo de impulso, e o módulo de elasticidade dinâmico (MOE\textsubscript{d}) foi determinado a partir da...
frequência do primeiro modo de vibração de flexão. Um total de 20 vigas de madeira foram obtidas de toras de *Pinus* spp. recém cortadas. Foram utilizadas toras com alto teor de umidade inicial, com dimensões nominais de 5 cm × 10 cm × 200 cm (espessura × largura × comprimento). Observou-se que o MOEₚ também é afetado pelo teor de umidade de acordo com o modelo de regressão não linear obtido. Houve mudanças significativas no módulo de elasticidade dinâmico (MOEₚ) para os teores de umidade (M) abaixo de 25% (próximo ao ponto de saturação da fibra - FSP).

**Palavras-chave:** Ensayo de flexión; Ensayo no destructivo; Madeira de *Pinus* spp.; Frecuencia; Saturación das fibras.

**1. Introduction**

Wood elements are widely used in civil construction for being a renewable material from natural forest or reforestation. Their use in rural and urban construction is a global trend to move towards cleaner production with an environmentally friendly raw material (Souza et al., 2018). Therefore, it is essential to know its mechanical properties to recommend its correct use and any procedure aimed at reducing the number of mechanical tests (need of several samples and destructive tests) is greatly desirable (Almeida et al., 2020).

The bending static modulus of elasticity (MOEₚ) can be evaluated by static tests (destructive technique - DT) or transverse vibration dynamic tests (non destructive techniques - NDT), among others. Some of the advantages of transverse vibration tests are shorter execution times, repeatability and possibility to obtain the MOEₚ considering the shear stress-induced deformation.

The transverse vibration test technique has shown good results to estimate the MOEₚ of wood elements (Carreira et al., 2012, 2017; Carreira & Segundinho, 2012; Medeiros Neto et al., 2016; Segundinho et al., 2012, 2013). The transverse vibration NDT techniques can be used successfully as part of an automated structural lumber grading system (França et al., 2018).

The differential equation 1 describes the transverse displacement (v) of a wooden beam in free transverse vibration over time (t), neglecting the shear effects (Clough & Penzien, 2003; Timoshenko, 2007). Equation 1 is valid for wooden beams whose length is much greater than the height of the cross section, in which case the shear stress effects can be neglected (Cho, 2007).

\[
m(x) \frac{\partial^2 v(y,t)}{\partial t^2} + \rho \frac{\partial^2}{\partial y^2} \left[ MOE_{st} \cdot J \cdot \frac{\partial^2 v(y,t)}{\partial y^2} \right] = 0
\]

In which: \( m \) - mass per unit length; \( x \) and \( y \) - coordinates in the longitudinal and transverse directions, respectively; \( t \) - time; \( v \) - transverse displacement and \( MOE_{st} \) - static modulus of elasticity.

Equation 2 is subsequently obtained from equation 1 using the separation of variables method and applying the boundary conditions; it relates the dynamic modulus of elasticity (MOEₚ) of the wooden beam with frequency \( f \) (Hz) of the first bending mode under the free-free boundary condition (Clough & Penzien, 2003).

\[
MOE_{d} = \frac{f^2 \cdot A \cdot ρ \cdot A}{12 \cdot J \cdot \rho \cdot A}
\]

In which: \( MOE_{d} \) - dynamic modulus of elasticity (Pa); \( f \) - frequency (Hz); \( L \) - wooden beam length (m); \( ρ \) - apparent wood density (kg m⁻³); \( A \) - area (m²); \( J \) - moment of inertia of the cross section (m⁴).

There are several references in the literature regarding the effect of moisture content on measuring the bending MOEₚ.
with transverse vibration testing (França et al., 2018; Teixeira, 2016). In an experimental investigation of the moisture content effect on measurements of MOE\textsubscript{d} of wooden beams, were added extra weights (dead weights) distributed along the beam length to simulate the addition of free water (moisture content > fiber saturation point - FSP ≈ 30%) (Barrett & Hong, 2010). The authors found that the dynamic modulus of elasticity remains constant by adding load to simulate moisture contents above 30%.

 Corrections for the MOE\textsubscript{d,12} of spruce species were proposed to evaluate the moisture content influence on MOE\textsubscript{d} of wooden beams for moisture content below (equation 3) and above (equation 4) FSP, respectively (Unterwieser & Schickhofer, 2011).

\[
\text{MOE}_{d,12} = \frac{\text{MOE}_d}{1-0.0087(M-12)} \quad (3)
\]

\[
\text{MOE}_{d,12} \approx 1.15 \times \text{MOE}_{d,M} \quad (4)
\]

In which: MOE\textsubscript{d,12} - dynamic modulus of elasticity (Pa) for moisture content equal to 12%; MOE\textsubscript{d} - dynamic modulus of elasticity (Pa); M - moisture content (%).

Equation 5 indicates a correction for the MOE\textsubscript{st} value obtained by static tests when moisture content is less than or equal to 20% in order to normalize to the standard moisture of 12% (Associação Brasileira de Normas Técnicas. NBR 7190-I: Projeto de estruturas de madeira: Critérios de dimensionamento, 2022). The value of the MOE\textsubscript{st} is assumed to remain constant for moisture content greater than 20%.

\[
\text{MOE}_{st,12} = \text{MOE}_{st} \times [1 + 0.02 \times (M - 12)] \quad (5)
\]

In which: MOE\textsubscript{st,12} - dynamic modulus of elasticity (Pa) for moisture content equal to 12%; MOE\textsubscript{st,M} - dynamic modulus of elasticity (Pa); M - moisture content (%).

Information on the advantages of the rational use of wooden elements in the construction of structural design and their physical-mechanical properties are essential to develop research mainly aimed at the consumer market (Almeida et al., 2020).

Thus, the objective of this article is to present an experimental investigation about the influence of the moisture content on the dynamic modulus of elasticity (MOE\textsubscript{d}) of wooden beams by transverse vibration tests.

2. Material and Methods

A total of 20 sawn wooden beams of Pinus spp. with nominal dimensions of 5.0 cm × 10.0 cm × 200.0 cm (thickness × width × length) extracted from freshly cut logs were used. The beams were kept in an acclimatized room (temperature of 21.4 °C - 24.6 °C and relative humidity - RH of 55% - 65%). The moisture content was estimated by the weight of each wooden beam calculated daily, and the initial moisture content (M) was estimated from specimens taken from each beam. The static bending and transverse vibration tests were performed at approximately every 5% moisture content decrease until reaching equilibrium moisture content (M) equal to 12%. The oven-dry weight of the beams was obtained, from which the actual moisture contents during the tests were obtained.

The static bending test was done with a concentrated load at the half length of a simply supported wooden beam attending (American Society for Testing and Materials. D-198: Standard test methods of static tests of lumber in structural sizes, 2009) for the flexural test, with a span (S) of 185 cm. The static modulus of elasticity (MOE\textsubscript{st}) was calculated considering the interval between the vertical displacements 4 mm and 8 mm less than S/200 (Associação Brasileira de Normas Técnicas. NBR 7190-I: Projeto de estruturas de madeira: Critérios de dimensionamento, 2022).

The transverse vibration tests were conducted with excitation in the vertical plane. The wooden beams were suspended by two nylon cords (0.7 mm in diameter), positioned at coordinates 0.224 and 0.776 of the beam length (McConnell & Varoto, 2008), corresponding to the nodal points of the first bending vibration mode of a beam in free-free suspension (Figure 1) (Chui, 1991). The cords were tied to low rigidity springs so that the rigid body mode frequency was less than 10% of the natural
frequency of the first bending mode vibration, thereby ensuring the free suspension behavior (McConnell & Varoto, 2008).

**Figure 1.** Schematic of the transverse vibration test. L: wooden beam length.

An accelerometer (Endevco, 7254A-100) was fastened at one end of the wooden beam with beeswax. An impulse hammer (B & K, 8206-002) produced excitement. The hammer and accelerometer signals were sent to a signal conditioner and converted into digital signals by a USB 6009 board (National Instruments). The Frequency Response Function (FRF) was calculated using the average of 10 spectra (He & Fu, 2001). The modal parameters were identified with the Modal-Id program which performs the multimodal identification by the polynomial ratio method obtained according to the same authors. The dynamic modulus of elasticity was calculated using equation 2.

A dimensionless coefficient (C) was calculated (equation 6) which relates the dynamic modulus of elasticity in the moisture content of the test (MOEd) with the static modulus of elasticity for moisture content equal to 12% (MOEd,12, equation 3) in order to assemble all the dynamic test results of *Pinus* spp. wooden beams in a single graph. In addition, the relative moisture content (Mrel) was obtained as equation 7.

\[
 MOE_d = C \cdot MOE_{d,12} \tag{6}
\]

\[
 M_{rel} = M / 12 \tag{7}
\]

In which: MOEd - dynamic modulus of elasticity (Pa); MOEd,12 - dynamic modulus of elasticity (Pa) for moisture content equal to 12%; Mrel - relative moisture content; M - moisture content (%); C - dimensionless coefficient.

**3. Results and Discussion**

**3.1 Influence of moisture content**

A graph of the dimensionless coefficient (C) values versus the relative moisture content (Mrel) was developed based on equations 6 and 7 (Figure 2). It is observed that there was an agglomeration of the C values to relative moisture content (Mrel) < 2%, and a decrease of the same. A constant dispersion of C values is observed from (Mrel) ≥ 2, with mean values around 0.9.

Thus, the dynamic modulus of elasticity (MOEd) remains virtually constant for moisture content above 25% (Mrel greater than 2) (Figure 2). In *Larix gmelinii* wood, the MOEd decreased with the increase in moisture content (Cheng et al., 2020). However, the trends began to slow down when the moisture content was higher than the FSP.
A preliminary statistical analysis of the relation between $C$ and $M_{rel}$ considering moisture content below 25% showed the presence of five outliers, all identified from only one beam. Discarding the data for this beam, the regression curve between $C$ and $M_{rel}$ was obtained for $M_{rel} \leq 2.08$ (Figure 3) with a good coefficient of determination ($R^2$ equal to 0.72).

By replacing the value of $C$ obtained in the regression analysis (Figure 3) in equation 6, the estimated value of the $MOE_{st,12}$ for moisture content equal to 12% ($MOE_{st,12}$) can be obtained as a function of the $MOE_d$ and moisture content $\leq 25\%$, equation 8.

$$MOE_{st,12} = \frac{MOE_d}{0.84 + 0.56 \cdot e^{0.083 \cdot M}}$$  \hspace{1cm} (8)$$

In which: $MOE_{st,12}$ - static modulus of elasticity (Pa); $MOE_d$ - dynamic modulus of elasticity (Pa); $M$ - moisture content (%).

Figure 4 shows the least squares fitted function for $C$ values for relative moisture content ($M$) for values higher than 25% ($M_{rel} > 2.08$). It indicates that there was low correlation with a low coefficient of determination ($R^2$ equal to 0.04) between the

---

Figure 2. Relationship between dimensionless coefficient ($C$) and relative moisture content ($M_{rel}$).

![Image of Figure 2](image)

Source: Authors.

Figure 3. Nonlinear regression between dimensionless coefficient ($C$) and relative moisture content ($M_{rel}$) for moisture $\leq 2.08$.

![Image of Figure 3](image)

Source: Authors.
MOE_d and the moisture content, which indicates that the dynamic modulus of elasticity remains constant or without a definite trend. This should remain for values above the FSP.

**Figure 4.** Linear regression between dimensionless coefficient (C) and relative moisture content (M_{rel}) for values more than 2.08 (M > 25%).

\[
\hat{Y} = 0.8841 + 0.0031 \cdot M_{rel} \\
R^2 = 0.0558
\]

Source: Authors.

Next, the theoretical frequency of vibration was calculated from equation 2 to verify this assumption for moisture contents above 25%. The calculation admitted that the MOE_d and the cross-section dimensions remain constant and equal to the values measured for moisture content equal to 25%. The wooden beam probably absorbs moisture near the FSP or capillary condensation caused by lowering the water vapor pressure, and the stress wave reflects and refracts between the wood fiber and the resulting water molecule. These phenomena affect the stress wave propagation velocity (Cheng et al., 2020).

Figure 5 shows results of the linear regression between the experimentally measured frequency of vibration (f_{experimental}) and the theoretical frequency (f_{theoretical}), indicating that there was high correlation with a high coefficient of determination (R^2 equal to 0.99).

**Figure 5.** Linear regression between frequency of experimental vibration (f_{exp}) and theoretical frequency (f_{theo}) for moisture content > 25%.

\[
\hat{Y} = 0.9077 + 0.9829 \cdot f_{theoretical} \\
R^2 = 0.9933
\]

Source: Authors.
If the cross-section dimensions do not change significantly for moisture content above 25%, it can be concluded that the MOE did not change significantly for moisture contents above the FSP, assumed as constants for moisture contents above than 25%. Theoretical and experimental results for *Larix gmelinii* wood showed that when the moisture content did not reach the FSP, the wood properties sharply decreased with increased moisture (Cheng et al., 2020). However, the trends began to slow down when the M was higher than the FSP.

We can obtain equation 9 by replacing moisture equal to 25% in equation 8, which corrects the MOE value obtained for M greater than 25%, for the MOE value for moisture content equal to 12% (MOE).

\[
\text{MOE}_{\text{st,12}} = 1.1 \times \text{MOE}_d
\]  

In which: MOE - modulus of elasticity (Pa); MOE - dynamic modulus of elasticity (Pa); M - moisture content (%).

### 3.2 Determining the modulus of elasticity for moisture at 12%

Correction methods shown in the literature (Clough & Penzien, 2003; Timoshenko, 2007), and the non-linear regression model by least-squares fit were evaluated (Figure 3) in order to compare the correction methods, the percentage (residual) error between the measured and estimated MOR with each method, both with M equal to 12%. Figure 6 and Figure 7 show the graphs obtained for M ≤ 25% and for M > 25%, respectively.

It is observed that the adjusted model for moisture contents below 25% results in lesser intensity residues, with the average of residues at around zero. Differently from that indicated in the literature (Associação Brasileira de Normas Técnicas. NBR 7190-1: Projeto de estruturas de madeira: Critérios de dimensionamento, 2022; Unterwieser & Schickhofer, 2011) where the correction equations result in larger residuals with the same order of magnitude.

On the other hand, the residues for moisture contents greater than 25% obtained by the corrections equations proposed in the same literature are equivalent.

**Figure 6.** Residues obtained in the estimation of MOE from dynamic modulus of elasticity (MOE) for moisture content (M) ≤ 25%.

**Figure 7.** Residues obtained in the estimation of MOE from dynamic modulus of elasticity (MOE) for moisture content (M) > 25%.

Source: Authors.
4. Conclusion

Considering the correlations obtained and the residual analysis, it can be concluded that the non-linear regression adjusted model resulted in more accurate estimates for the wood samples studied and that the MOE$_d$ is also affected by moisture content according to the non-linear regression model.

It was concluded that the equations proposed to calculate the MOE$_{d,12}$ in the moisture content equal to 12% results in values more than those obtained using the equation recommended by the Brazilian Standard. That is, the MOE$_d$ for moisture content (M) equal to 12% can be estimated for moisture content below 25%.

It can be assumed that the MOE$_d$ remains constant for moisture contents above 25%. Thus, the results of dynamic tests performed with moisture contents above 25% will not be affected by moisture content variation for the Pinus sp. wood species studied for structural classification purposes.

It is suggested that this type of research is also expanded to native species, as to other exotic species, because in this way it will be possible to know their behavior in determining the dynamic modulus of elasticity due to humidity variation.

Acknowledgements

This study was financed in part by the Coordenação de Aperfeiçoamento de Pessoal de Nível Superior - Brasil (CAPES) - Finance Code 001 and in part by the Conselho Nacional de Desenvolvimento Científico e Tecnológico - Brasil (CNPq).

References


