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Uma aplicação do algoritmo *Particle Swarm Optimization* (PSO) com dados diários de precipitação em Campina Grande, Paraíba, Brasil

An application of Particle Swarm Optimization (PSO) algorithm with daily precipitation data in Campina Grande, Paraíba, Brazil

Una aplicación del algoritmo *Particle Swarm Optimization* (PSO) con datos de precipitación diaria en Campina Grande, Paraíba, Brasil

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Resumo

Estudamos a precipitação diária na cidade de Campina Grande estimando os parâmetros das distribuições Gamma, Log-Normal e Weibull comparando a Otimização do algoritmo

Particle Swarm Optimization (PSO) versus a Estimativa de Máxima Verossimilhança (MLE) para analisar e também entender o comportamento da precipitação diária. em Campina Grande. Na maioria dos casos, os resultados obtidos mostraram evidências de que o algoritmo PSO é uma técnica eficiente e robusta. Não obstante, o algoritmo também apresenta uma eficiente alternativa para a estimativa de parâmetros devido à sua rápida convergência.
Palavras-chave: PSO; Precipitação; Campina grande.

Abstract

We study the daily precipitation in the municipality of Campina Grande, estimating the parameters of Gamma, Log-Normal, and Weibull distributions. To evaluate the parameter estimators, we compared the Particle Swarm Optimization (PSO) versus Maximum Likelihood Estimation (MLE) to analyze and understand the behaviour of the daily precipitation in Campina Grande. In most cases, our results show evidence that the PSO algorithm is an efficient and robust technique. Notwithstanding, the algorithm also presents an efficient parameter estimation due to its fast convergence.
Keywords: PSO; Precipitation; Campina grande.

Resumen

Estudiamos la precipitación diaria en la ciudad de Campina Grande al estimar los parámetros de las distribuciones Gamma, Log-Normal y Weibull comparando la optimización de la optimización de enjambre de partículas (PSO) versus la estimación de máxima verosimilitud (MLE) para analizar y también comprender el comportamiento de la precipitación diario. en Campina Grande En la mayoría de los casos, los resultados obtenidos mostraron evidencia de que el algoritmo PSO es una técnica eficiente y robusta. Sin embargo, el algoritmo también presenta una alternativa eficiente para la estimación de parámetros debido a su rápida convergencia.
Palabras clave: Precipitación; PSO; Campina grande.

1. Introduction

A crucial point to analyze rainfall data is strongly dependent on its distribution pattern. In the fields of meteorology and climatology, it is common to establish an appropriate probability distribution that contributes to a proper fit for daily rainfall.

A variety of studies have been handled in Brazil and abroad on rainfall analysis and the best fit probability distribution function such as normal, Lognormal, Gumbel, Weibull, Pareto and some generalization were identified (Ben-Zvi, 2009; Carneiro et al., 2016; Franco et al., 2014; Papalexiou, Koutsoyiannis & Makropoulos, (2013); Sugahara, Da Rocha & Silveira, 2009).

The standard scheme for selecting the best distribution law for rainfall is to

(i) try some of many, a priori chosen, parametric families of distributions,
(ii) estimate the parameters according to one of many existing fitting methods, and
(iii) choose the one best fitted according to a specific fitting test.

As stated in Beskow et al. (2015), statistical inference techniques have a considerable influence on the adjustment quality of a probabilistic model. There are numerous statistical inference methods, such as the method of moments, maximum likelihood, (ordinary) least squares.

Let X a random variable with size n and the density function with a parameter θ. The likelihood function is given by

$$L(X|\theta) = \Pi_{i=1}^{n} f(x_i|\theta), \qquad (1)$$

Wherein is defined as the parameter vector with dimension m. The goal is to find a vector that maximizes Eq.(1). A variety of gradient-based and non-gradient optimization methods can be tested. The gradient-based optimization method is unstable to the initial values' determination, making it possible to end up at locally optimum solutions. Hence, it is mandatory to apply a heuristics-based optimization method (Handoyo et al., 2017). Heuristic optimization methods such as simulated annealing (SANN) and PSO have been used to the likelihood function of statistical distributions (Abbasi et al., 2006; Örkcü et al., 2015). Do Nascimento et al., (2020a) compared the adjustments made between the Weibull distribution with the Method of Moments, with the Maximum Likelihood Estimation and with the Particle Swarm Optimization algorithm, as well as the Lognormal and Weibull adjustments both with the PSO for a wind dataset in the municipality of Petrolina. Do Nascimento et al. (2020b) utilized the PSO algorithm to a better understanding of financial markets and assisting market participants in the definition of trading strategies aimed at the minimization of financial losses.

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The statistical literature is filled with illustrations in which the Gamma, Lognormal and Weibull distributions are used entirely to explain real phenomena (Yan et al., 2002; Cho, Bowman & North, 2004; Wong et al., 2010; G. R. 2004). The paper is organized as follows. In the following section, we present the dataset and methodology, in the next section, we display the results and the discussion, and finally, the conclusions are drawn.

2. Methodology

The methodology used refers to a case study of the modelling of daily precipitation collected from a rain gauge station in Campina Grande, PB, to find the best probabilistic model to describe the data collected. Thus, we compare two methods for estimating the parameters of the probabilistic models used through some error measures: The maximum likelihood method and the Particle Swarm Optimization algorithm. Therefore, our study deals with qualitative and quantitative work (Pereira et al., 2018). The Particle Swarm Optimization algorithm has been successfully applied in several applications regarding the estimation of parameters in hydrological models (Bardolle et al., 2014; Jakubcová, Máca & Pech, 2015; Taormina & Chau, 2015).

2.1 Data

The municipality of Campina Grande lies in the eastern part of the Borborema plateau in Paraíba State, Brazil, at 7°14'S, 35°54' W. It has the climate type Aw'i, conforming to the Köppen climate classification and is classified as dry sub-humid. The rainy season occurs between March to July, and the average precipitation is about 800 mm (1974-2004). The average annual maximum temperature is 28.7° C and a minimum of 19.8° C, which ranges slightly throughout the year (Pereira et al., 2016). The data on daily precipitation in Campina Grande from February 2, 1963, to December 21, 2017, were collected from the official National Institute of Meteorology (INMET) website http://www.inmet.gov.br.

2.2 Particle Swarm Optimization

The computational algorithm Particle swarm optimization (PSO) was introduced by Eberhart, & Kennedy (1995) and Sammut & Webb (2011). PSO is a technique derived from the mutual behaviour of bird flocks. PSO, also known as an evolutionary computation technique used as an optimizer, is characterized as a population-based, self-adaptive search optimization technique (Fukuyama, 2008). According to Örkcü et al., (2015), PSO resides on a set of solutions (particles) named population. Each solution consists of a set of parameters and performs a point in multidimensional space. In most cases, the essential idea of PSO can be described as follows: firstly, a swarm of particles keeps moving around in a search-space according to a few simple formulae. A suitable solution will eventually come out at the end. Assume the search space is a D-dimensional space and the number of particle swarm is m, then the particle i of the swarm can be expressed by a d-dimensional vector as $x_i = (x_{i1}, x_{i2}, \dots, x_{id})$; and the speed of the particle expressed as $v_i = (v_{i1}, v_{i2}, \dots, v_{im})$, $i = 1,2, \dots, n$. The following algorithm can be described as (Ren et al., 2014):

I. A group of m particles with positions and velocities have to be randomly initialized;
II. Calculate the fitness value of each particle;
III. Calculate the position of best fitness value of each particle from its historical movement: (*pbest*);
IV. Calculate the position of best fitness value of all particle from global historical movement: (*gbest*);
V. Update particles' position and speed using the two following formulas.

$$v_i^{t+1} = \omega^t v_i^t + c_1\tau_1(pbest^t - x_i^t) + c_2\tau_2(gbest^t - x_i^t) \tag{2}$$

$$x_i^{t+1} = x_i^t + v_i^{t+1} \,. \tag{3}$$

Wherein v_i^t is the velocity of particle i at iteration t, and x_i^t represents the position of particle i at iteration t, respectively. τ_1 and τ_2 are two uniform random numbers from $[0,1]$, and c_1 and c_2 are the learning factors, also called acceleration constants as they regulate how far a particle can move in a single iteration. The variable τ_t is inertia weight at iteration t described by Eq.(4) below. A larger inertia weight produces the global exploration and a smaller inertia weight tends to ease the local exploration to fine-tune the current search area.

$$\omega^t = \omega_{max} - (\omega_{max} - \omega_{min}) \times \frac{iter}{itmax} \tag{4}$$

Here, ω_{max} and ω_{min} represent the maximum and minimum inertia weight, respectively; $itmax$ is the maximum of iteration number.

VI. If the last condition is not satisfied, loop back to step II again, the end condition is usually a fixed $itmax$ value or adjusts fitness value.

2.3 Performance of estimation accuracy

In order to analyze the efficiency of the estimation, various performance measurement methods have been used; however, they have not been recognized as a global standard method. Therefore, it is significant to widely cover some performance metrics to understand the algorithm characteristics (Chai & Draxler, 2014; Yaseen et al., 2019). The metrics which were used in this study are given by

$$MAE = \frac{1}{n}\sum_{i=1}^{n}(y_i - \hat{y_i}) \tag{5}$$

$$RMSE = \frac{1}{n}\sqrt{\sum_{i=1}^{n}(y_i - \hat{y_i})} \tag{6}$$

$$MAPE = \frac{1}{N}\sum_{i=1}^{n}\left|\frac{y_i - \hat{y_i}}{y_i}\right| \times 100\% \tag{7}$$

$$RAE = \frac{\sum_{i=1}^{n}|p_i - a_i|}{\sum_{i=1}^{n}|\bar{a} - a_i|} \tag{8}$$

2.4 Gamma Distribution

The Gamma distribution, as well as the exponential distribution and the Lognormal distribution, is one of the largest generally used distributions for rainfall data (Wilks, 1998). The PDF and CDF of the Gamma distribution are given, respectively, by

$$f(x) = \frac{1}{\beta\Gamma(\alpha)}\left(\frac{x}{\alpha}\right)^{\alpha-1}\exp\left(-\frac{x}{\beta}\right) \tag{9}$$

$$F(x) = \int_0^x f(x)dx = \int_0^x \frac{1}{\beta\Gamma(\alpha)}\left(\frac{x}{\alpha}\right)^{\alpha-1}\exp\left(-\frac{x}{\beta}\right)dx, \tag{10}$$

$\Gamma(\alpha)$ is the Gamma function, and the two parameters α and β represent the scale and shape, respectively.

2.5 Weibull Distribution

The Weibull distribution, which can be considered as a generalization of the exponential distribution, is a typical model in hydrology (Heo, Salas & Boes, 2001) and its PDF and is given by

$$f(x) = \frac{\alpha}{\beta}\left(\frac{x}{\alpha}\right)^{\alpha-1}\exp\left(-x/\beta\right)^{\alpha}, \tag{11}$$

where $\beta>0$ is a scale parameter, while $\alpha>0$ is the shape parameter, which controls the tail's asymptotic behaviour.

2.6 Lognormal Distribution

The Lognormal distribution is part of the subexponential class and is treated as a heavy-tailed distribution (Papalexiou, Koutsoyiannis & Makropoulos, 2013). The Lognormal distribution considers that the logarithms of the data are normally distributed. The PDF of the Lognormal distribution is given by

$$f(x) = \frac{1}{x\sigma\sqrt{2\pi}}\exp\left\{-\frac{\left[\ln\left(\frac{x}{e^{\mu}}\right)\right]^{2}}{2\sigma^{2}}\right\}, x > 0 \tag{12}$$

wherein μ and σ are the mean and standard deviation of the logarithmically transformed variables, respectively.

3. Results and Discussion

The time series of daily precipitation used for this study, from February 2, 1963, to December 21, 2017, is shown in Figure 1.

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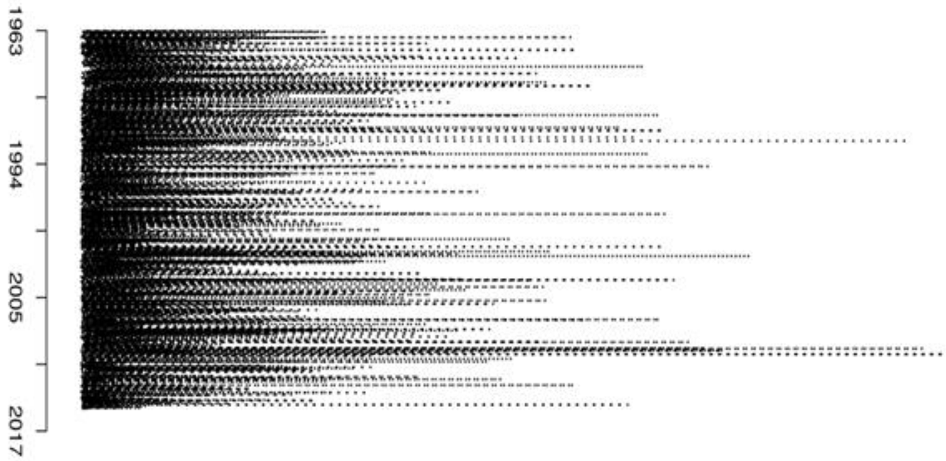
Figure 1. Daily precipitation series from 1963 to 2017.

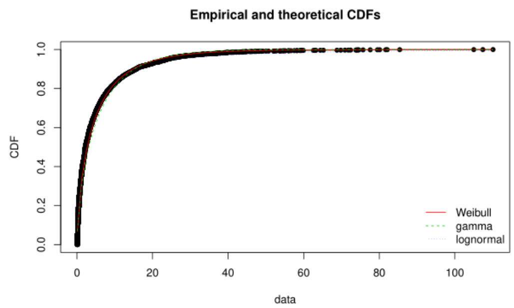


Source: Prepared by the authors.

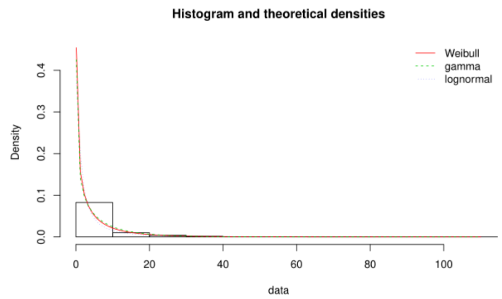
It can be observed from Figure 1 that the precipitation series is stationary. We performed the Dickey-Fuller test, which presented a p-value $< 0{,}01$ that guarantees the stationarity of the series.

Figure 2 displays the fitting of distribution Gamma, Weibull and Lognormal by the empirical and theoretical CDFs (a), histogram and theoretical densities (b), quantile function (c) and probability function (d) estimated by maximum likelihood for daily precipitation series from 1963 to 2017 in Campina Grande, Brazil.

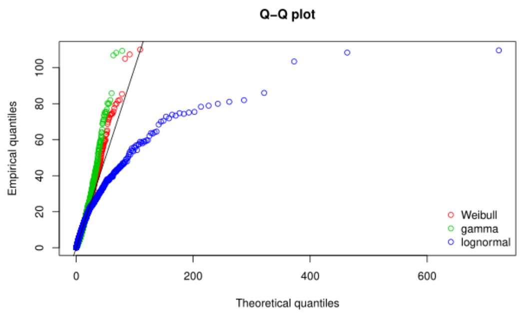
Figure 2. Fitting of distribution Gamma, Weibull and Log-Normal via empirical and theoretical CDFs (a), histogram and theoretical densities (b), quantile function (c) and probability function (d), respectively.



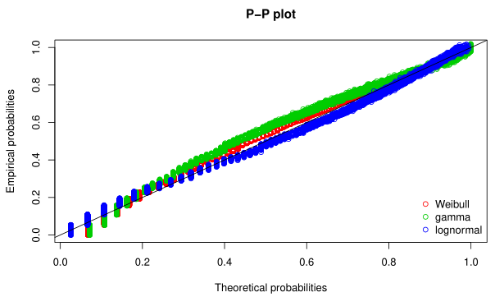
(a)



(b)



(c)



(d)

Source: Prepared by the authors.

It can be noticed that Weibull presented better behaviour when observed theoretical quantiles and theoretical probabilities, Figure 2 (c)-(d), respectively. Particularly, there was no difference among the distributions regarding empirical and theoretical CDF displayed in Figure 2 (a).

Figure 3 exhibits the behaviour of PSO based on Gamma, Lognormal, and Weibull simulations using 30 simulations with 300 iterations for each simulation to verify the fitness function's adjustment.

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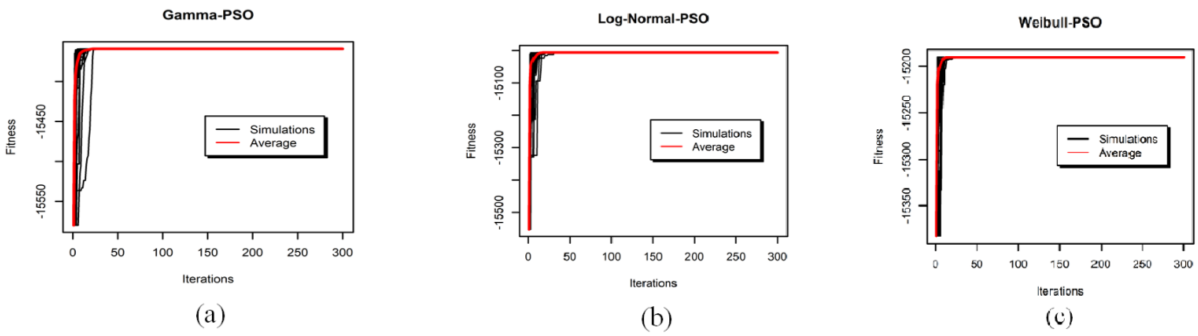
Figure 3. PSO behavior via the evolution of fitness in iterations for three distributions simulations and average.

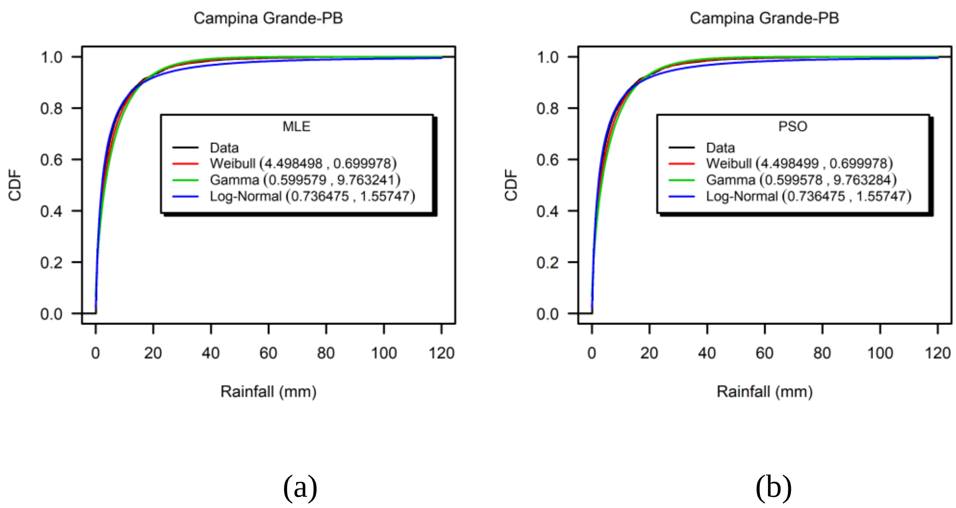


Source: Prepared by the authors.

One can see from Figure 3 that Weibull-PSO presents the best fit (Figure 3 (c)) in opposition to Log-Normal (Figure 3 (b)), and Gamma-PSO (Figure 3 (a)) which displays a higher variability.

Figure 4 displays the values of estimates of the parameters for Gamma, Lognormal and Weibull for CDF and PDF(Density), respectively.

Figure 4. The behaviour of distributions' estimates via MLE (a) and PSO (b).



Source: Prepared by the authors.

It is noticeable from the Figure 4 that the values of the estimates are practically identical in both cases, that is, the behaviour of distributions estimates via MLE (a) and PSO (b) did not present numerical difference.

Table 1 shows the criterion utilized to attest the goodness-of-fit to verify the comparison between PSO and MLE.

Table 1. Goodness-of-fit for distributions used in this study.

			Goodness of fit			
Criteria	Gamma	Gamma-PSO	Lognormal	Lognormal-PSO	Weibull	Weibull-PSO
AIC	30722,139	30722,139	30016,398	30016,398	30383,416	30383,416
AICc	30722,141	30722,141	30016,400	30016,439	30383,418	30383,457
BIC	30735,461	30735,461	30029,721	30023,806	30396,739	30390,824

Source: Prepared by the authors.

Results from Table 1 show that according to the criterion's values, there was no significant difference because the estimates were somewhat similar, except in AICc for Gamma, in which PSO presented a slightly better estimation.

Table 2 shows the performance metrics of estimation accuracy used to compare the MLE versus PSO.

Table 2. Performance metrics of estimation accuracy.

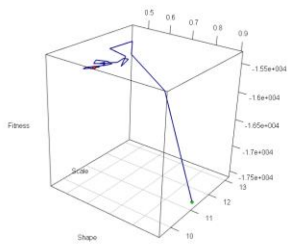
Distributions	MAE	MAPE	RAE	RMSE
Gamma	0,244572	11,399409	0,171563	0,049018
Gamma-PSO	0,244572	11,399462	0,171564	0,049018
Lognormal	0,243236	10,738153	0,094724	0,026812
Lognormal-PSO	4,682288	206,709825	0,094725	0,117637
Weibull	0,243518	7,412007	0,098225	0,028081
Weibull-PSO	4,687712	142,681583	0,098225	0,123205

Source: Prepared by the authors.

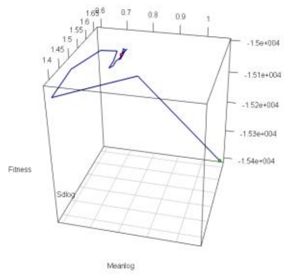
One can see that Gamma-PSO and Gamma did not present variation, which means both methods are efficient. However, Lognormal-PSO and Weibull-PSO presented the worst results compared to Lognormal and Weibull; that is, in both cases, the estimative through PSO was not suitable.

Figure 5 shows the PSO 3-D graphics for calculating the position of the best fitness (*pbest*) in which it is possible to observe how the swarm particles move over the search space using a 3-D perspective tool.

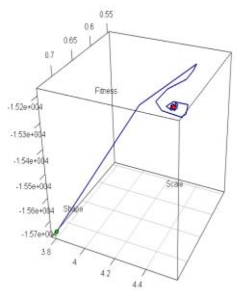
Figure 5. A PSO 3-D perspective for calculating the position of the best fitness (*pbest*) to Gamma (a), Log-Normal(b) and Weibull(c), respectively.



(a)



(b)



(c)

Source: Authors.

One can see in Figure 5 that the particle is evolving in the search space, looking for the global minima position.

4. Final Considerations

In this work, we investigate the estimates for Gamma, Log-Normal and Weibull distributions for daily rainfall using PSO and MLE. We analyzed the results and found that although PSO's algorithm provides results that are not significantly different, but with particular behaviours. The simulated estimates via PSO are the fastest type in getting a robust convergence.

We suggest further study, implementation of PSO for more rainfall stations with daily & monthly rainfall data, and other features like distance, altitude, relief, and climatic characteristics to validate the performance of PSO in solving MLE estimates for others statistical distributions. Essentially, the algorithm PSO is valuable in addressing the MLE for estimating the parameter of the distributions utilized.

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Tiago Almeida de Oliveira – 20%

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