Modelagem de internações por doenças respiratórias em função das funções de distribuição de probabilidade

Modeling of hospital admissions for respiratory diseases as a function of probability distribution functions

Modelado de ingresos hospitalarios por enfermedades respiratorias en función de las funciones de distribución de probabilidad

Resumo

Objetivo: Analisar os ajustes das distribuições de densidade de probabilidade weibull, gama, normal e logística da série histórica de hospitalizações por doenças respiratórias (pneumonia infantil e adulto) de 2011 a 2015, em Campo Grande, MS. Métodos: Os parâmetros de forma
e escala das distribuições foram determinados para verificar a qualidade do ajuste dos dados. Resultados: Quatro funções de densidade de probabilidade (Tabela 2) foram ajustadas e os testes R2, MAE, RSME, MAPE foram utilizados para verificar a melhor função de densidade para dados de hospitalização. Conclusão: O melhor ajuste foi a distribuição gama; a distribuição pode ser usada como uma distribuição alternativa que descreve adequadamente os dados de internações por doenças respiratórias em Campo Grande.

**Palavras-chave:** Internação hospitalar; Pneumonia; Modelagem; Probabilidade; Criança e adultos.

**Abstract**

Objective: To analyze the adjustments of the weibull, gamma, normal and logistic probability density distributions of the historical series of hospitalizations for respiratory diseases (childhood and adult pneumonia) from 2011 to 2015, in Campo Grande, MS. Methods: The shape and scale parameters of the distributions were determined to verify the quality of the data fit. Results: Four probability density functions (Table 2) were fitted and the R², MAE, RSME, MAPE tests were used to verify the best density function for hospitalization data. Conclusion: The best fit was the Gamma distribution; the distribution can be used as an alternative distribution that adequately describes the data on hospital admissions for respiratory diseases in Campo Grande.

**Keywords:** Hospital admission; Pneumonia; Modeling; Probability; Child and adults.

**Resumen**

Objetivo: analizar los ajustes de las distribuciones de densidad de probabilidad weibull, gamma, normal y logística de la serie histórica de hospitalizaciones por enfermedades respiratorias (neumonía infantil y adulta) de 2011 a 2015, en Campo Grande, MS. Métodos: se determinaron los parámetros de forma y escala de las distribuciones para verificar la calidad del ajuste de los datos. Resultados: se ajustaron cuatro funciones de densidad de probabilidad (Tabla 2) y se utilizaron las pruebas R2, MAE, RSME, MAPE para verificar la mejor función de densidad para los datos de hospitalización. Conclusión: el mejor ajuste fue la distribución Gamma; La distribución se puede utilizar como una distribución alternativa que describa adecuadamente los datos sobre ingresos hospitalarios por enfermedades respiratorias en Campo Grande.

**Palabras clave:** Ingreso hospitalario; Neumonía; Modelado; Probabilidad; Niños y adultos.
1. Introduction

Several studies of adjustment of probability density distribution or probability estimates using theoretical models of probability in relation to a historical series of data have been developed, highlighting the benefits in the planning of activities that minimize the risks, among which can be cited: precipitation (Catalunha, et al., 2002; Dallacort, et al., 2011; Hartmann, et al., 2011; Rodrigues, et al., 2014; Kist & Virgens Filho, 2015), air temperature (Assis, et al., 2018; Araújo, et al., 2010), concentration of pollutant gases (Souza, et al., 2018a; 2018b), for the historical series of hospital admissions for respiratory diseases there are no published works with this methodology.

The use of probability density functions is directly linked to the nature of the data to which they relate. Some have good estimation capacity for small numbers of data, others require a large number of observations. Due to the number of parameters of your equation, some can take different forms, being framed in a greater number of situations, that is, they are more flexible. Since respecting the aspect of data representativeness, the estimates of its parameters for a given region can be established as general purpose, without prejudice to the precision in the estimation of probability (Catalunha, et al., 2002).

Climate change has become one of the most serious environmental concerns for urban areas in recent decades. Several epidemiological studies in recent years have reported associations between high levels of climatic changes and increased rates of death and hospitalization for respiratory and cardiovascular diseases (Mahiyuddin, et al., 2013; Li, et al., 2016). Some epidemiological studies show that air pollution affects human health, even concentrations of air pollutants are below the air quality standards (Souza, et al., 2012).

Respiratory diseases and related mortality have been increasingly associated with exposure to climate change. Sensitive and vulnerable groups, such as pregnant women, children, the elderly, and those who already suffer from respiratory illnesses and other serious diseases, or from low-income groups, are especially affected by climatic variation. Studies have shown that the number of respiratory diseases in children and the elderly increases due to the higher concentrations of air pollution (Souza, et al., 2018c; Souza, et al., 2018d; Souza, et al., 2017; Santos, et al., 2017; Braga, et al., 1999; Braga, et al., 2001; Viegi, et al., 2009). According to these studies, children are more susceptible because they need twice the amount of air inhaled by adults, and the elderly are more affected because of their weakened immune and respiratory systems and have been exposed to a large amount of air pollution in all your life.
There are no published works (according to the best knowledge of the authors) on historical series of hospitalizations for respiratory diseases based on the methodology used in this research, developed by Souza, et al. (2019), which analyzed the adjustments of the Burr (Bu), Inverse Gaussian 3P (IG3P), Lognormal (LN), Pert (Pe), Rayleigh 2P (Ra 2P) and Weibull 3P (W3P) distributions in the series of hospitalizations for respiratory diseases (total hospitalizations). The parameters of the shape and scale of the distributions were determined and, to check the quality of the adjustment of the observation data, the quality adjustment tests (GOF) were applied: Kolmogorov-Smirnov, Anderson test - Darling Test, chi-square tests.

In the present study, which is the first hospital admission modeling study in Campo Grande, MS, Brazil, we performed a daily time series study of hospital admissions for respiratory conditions in Campo Grande, using probability distribution functions, Weibull, Gamma, Normal and logistics. The analysis was performed between people (children and adults) who contracted pneumonia.

2. Methodology

For the modeling of the data of hospital admissions in Campo Grande (MS) we used the functions of Weibull, Gamma, Normal and Logistic. Performance indicators are calculated by comparing observed values to predicted values. The observed values are the classified values of the observation data, while the predicted values are the values obtained from the adjusted distribution.

Data Collection

The city of Campo Grande - MS, (20° 27'16 "S, 54° 47'16" W, 650 m), is located on the plateau called Maracajú-Campo Grande, 150 miles from the start of the largest floodplain in the world, the Pantanal (139 111 km2), and an estimated population of 724,000 inhabitants. Souza & Granja (1997); using the Koppen's method, the climate in the region of Campo Grande, is the type with moderate temperatures ranging from 17.8 °C minimum, 29.8 °C maximum and average of 22.7 °C, with hot summer and well distributed rainfall, average relative humidity is 72.8%, found prevailing winds in East Campo Grande - MS, occurring in the North in months from January to December, with annual values resulted in 24% of East, 19.8% of North and 12.2% of Northeast, and the lulls represented 12% with an average speed.
of 3.1 m/s, and average monthly rainfall in 122.4 mm and annual average 1469 mm.

For the correlation of weather data with the aggravation of respiratory illnesses, hospitalization data were collected from the health agencies of SUS (Unified Health System) and Department of Informatics (DATASUS).

The available data came from the Hospital Information System of SUS (SIH / SUS), managed by the Ministry of Health, through the Department in Health Care, in conjunction with the State Departments of Health and the Municipal Health and processed by DATASUS at the Executive Department of the Ministry of Health.

All hospitalizations occurred in the period between January 1st, 2011 and December 31st, 2015, the diseases investigated were coded according to the International Classification of Diseases (CID) 10th Revision, Pneumology (J17). The subjects of this study were children (0-4 years) under of age and adult (5-60 years).

The geographical coordinates of the measurement location are shown in Table 1.

### Table 1. Geographical coordinates of the measurement site.

<table>
<thead>
<tr>
<th>City</th>
<th>Latitude</th>
<th>Longitude</th>
<th>Altitude (m)</th>
<th>Area (km²)</th>
<th>Measuring period</th>
</tr>
</thead>
<tbody>
<tr>
<td>Campo Grande</td>
<td>20°26'34&quot;S</td>
<td>54°38'47&quot;W</td>
<td>532</td>
<td>8118,4</td>
<td>Jan to Dec 2013/2015</td>
</tr>
</tbody>
</table>

Source: Authors (2020).

### Probability Distributions

In this study the effectiveness of twelve one-component probability distributions is evaluated. We have used the one-component parametric pdfs because our data presents a unimodal distribution. These twelve models have been selected among other one-component models due to their successful applications according to the literature. The used pdfs as well as their cumulative distribution functions (cdfs) and the number of their parameters (#) are presented in Table 2.
Table 2. List of used pdfs, their cdfs and number of parameters.

<table>
<thead>
<tr>
<th>Name</th>
<th>pdf</th>
<th>cdf</th>
<th>#</th>
<th>Nomenclature</th>
</tr>
</thead>
<tbody>
<tr>
<td>W2</td>
<td>( f(v) = \left( \frac{v}{c} \right)^{k-1} \exp\left( -\frac{v}{c} \right) )</td>
<td>( F(v) = 1 - \exp\left( -\frac{v}{c} \right) )</td>
<td>2</td>
<td>( c ) is the scale parameter and ( k ) is the shape parameter</td>
</tr>
<tr>
<td>N</td>
<td>( f(v) = \frac{1}{\sigma \sqrt{2\pi}} \exp\left( -\frac{(v - \mu)^2}{2\sigma^2} \right) )</td>
<td>( F(v) = \frac{1}{2} \left( 1 + \text{erf}\left( \frac{v - \mu}{\sigma \sqrt{2}} \right) \right) )</td>
<td>2</td>
<td>( \mu = \text{mean} ) (location parameter)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>( \sigma = \text{variance} ) (squared scale)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>( \alpha, \beta, \Gamma ) and ( \gamma ) are the shape parameter, scale parameter, the gamma function, and the incomplete Gamma function respectively.</td>
</tr>
<tr>
<td>G</td>
<td>( f(v) = \frac{v^{\alpha-1}}{\Gamma(\alpha) \beta^\alpha} \exp\left( -\frac{v}{\beta} \right) )</td>
<td>( F(v) = \frac{v^{\alpha-1}}{\Gamma(\alpha)} )</td>
<td>2</td>
<td>( \mu ) is the location parameter and ( s ) is the scale parameter.</td>
</tr>
<tr>
<td>L</td>
<td>( f(v) = \frac{\exp\left( -\frac{v - \mu}{\sigma} \right)}{\sigma \left[ 1 + \exp\left( -\frac{v - \mu}{\sigma} \right) \right]^2} )</td>
<td>( F(v) = \frac{1}{1 + \exp\left( -\frac{v - \mu}{\sigma} \right)} )</td>
<td>2</td>
<td>( \mu ) is the location parameter and ( s ) is the scale parameter.</td>
</tr>
</tbody>
</table>

Source: Authors (2020).

Estimation of distributions parameters

Several methods can be used to estimate the considered distributions parameters (22). However, the selection of effective distributions is more important compared to the selection of parameter estimation methods (Tizgui, et al., 2017). In this work, the Maximum Likelihood method (ML) is applied. This method has shown good results in several studies. It gives the values of the parameters which maximize the probability of obtaining the observed data.

The likelihood function (L) for a random sample of historical series of hospitalizations
for respiratory diseases $v_1, v_2, \ldots, v_n$ and theoretical probability density function (f) with $j$ parameters $\alpha_1, \ldots, \alpha_j$ is represented by equation:

represented by equation:

$$ L = \prod_{i=1}^{n} f(v_i; \alpha_1, ..., \alpha_j) $$

For each parameter $\alpha_i$, ML consists in estimating its value which maximizes the Likelihood function (L) by solving the following equation.

$$ \frac{\partial \log L}{\partial \alpha_i} = 0 $$

### Accuracy Tests

The accuracy tests (or goodness of-fit tests) are essential to compare the observed climate distributions with the predicted/ modelled distributions. The observed dataset is the values from the monitoring systems whereas the modelled datasets are obtained from the fitted distributions. In this study, two categories of goodness of-fit tests are used. $R^2$ and RMSE associated with pdf (which are calculated using the relative frequencies of the histogram and the predicted pdfs obtained by theoretical model), while MAE and MAPE associated with cdf (which are calculated using the empirical cumulative frequencies of observations and the predicted cdf of the studied models). These accuracy tests are based on histogram approach, in which the measurements are arranged in a relative frequency histogram with $N$ class intervals. The advantage of this approach is that it is less affected by individual measurements (Ouarda, et al., 2016). Their expressions are given below.

#### The coefficient of determination ($R^2$)

The coefficient of determination measures how much the variance of the measured data is explained by the theoretical model. In this work, $R^2$ is calculated using the class intervals, the relative frequencies of the histogram and the predicted pdfs obtained by theoretical model (24). $R^2$ is expressed as follow:
\[ R^2 = 1 - \frac{\sum_{i=1}^{N} (p_i - \hat{p}_i)^2}{\sum_{i=1}^{N} (p_i - \bar{p})^2} \]  

(3)

where \( \hat{p}_i \) is the predicted pdf at the at \( i^{th} \) interval, \( p_i \) is the relative frequency at the \( i^{th} \) class and \( \bar{p} = \frac{1}{N} \sum_{i=1}^{N} p_i \) (Jung, et al., 2017).

**Root Mean Square Error (RMSE)**

Since, it combines the bias and the dispersion, the root means square error is an important indicator for comparing the predicted with the observed values. The RMSE associated with probabilities in class intervals is given as:

\[ \text{RMSE} = \left[ \frac{1}{N} \sum_{i=1}^{N} (p_i - \hat{p}_i)^2 \right]^{\frac{1}{2}} \]  

(4)

Where \( p_i \) and \( \hat{p}_i \) are as defined in equation (3) above

**Mean Absolute Error (MAE)**

The mean absolute error is defined as the mean of the absolute errors derived from the observed and predicted values. The mathematical equation of MAE associated with cdf in class intervals is defined as:

\[ \text{MAE} = \frac{1}{N} \sum_{i=1}^{N} |F_i - \hat{F}_i| \]  

(5)

where \( \hat{F}_i \) is the theoretical cdf of the \( i^{th} \) measured wind speed and \( F_i \) is the empirical cdf of the measured wind speed at \( i^{th} \) time stage.

**Mean Absolute Percentage Error (MAPE)**
The mean absolute percentage error indicates the mean absolute percentage difference between the predicted and observed data. Basing on the histogram approach, the mean absolute percentage error associated with the cdfs is calculated as (Hu, et al., 2016):

\[
\text{MAPE} = \frac{1}{N} \sum_{i=1}^{N} \frac{|F_i - \hat{F}_i|}{F_i} \times 100\% \tag{6}
\]

**Ethical Considerations**

The present study is based on secondary, publicly available data, which do not constrain groups of populations and / or individuals in the presentation of the results found, ensuring the confidentiality of the information collected. Thus, the ethical aspects of research with human beings were respected, according to Resolution 466/2012 (Brasil, 2013).

**3. Results and Discussion**

**Descriptions of Hospitalizations of Respiratory Diseases**

Figure 1 illustrates a typical pattern of hospital admissions (morbidity) for respiratory diseases / pneumonia / children and adults, the daily averages for the months 2011-2015.

During the study period (January 1, 2011 to December 31, 2015), the number of hospitalizations for respiratory diseases was 609 (314 children and 295 adults, with a mean of 5 daily admissions, with a minimum of 2 and a maximum of 8). According to the data, a seasonal pattern was observed between the rainy season, the dry season and the transition period, especially in the quarters (April, May, June, July, August, and September), where the peak of hospitalizations corresponds to the dry season, low rainfall, relative humidity and minimum temperatures, and the period of highest burning rates in the state of Mato Grosso do Sul.
Figure 1. Hospital admissions (morbidity) due to respiratory diseases / pneumonia / in children and adults in Campo Grande, from 2011 to 2015.

Source: Authors.

Distributions of probability and its estimation of parameters

The parameters of the estimates of the tested distributions are presented in Table 3, these parameters are obtained using the ML in the MATLAB software.
### Table 3. Estimated parameters for the distributions studied.

#### Children

<table>
<thead>
<tr>
<th>Models</th>
<th>Estimates</th>
<th>$R^2$</th>
<th>MAE</th>
<th>RMSE</th>
<th>MAPE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Weibull</td>
<td>$a=5.74043$</td>
<td>0.526790437</td>
<td>0.050899654</td>
<td>0.070520265</td>
<td>50.49832</td>
</tr>
<tr>
<td></td>
<td>$b=3.63119$</td>
<td>0.95843169</td>
<td>0.064573933</td>
<td>0.097750451</td>
<td>17.75106</td>
</tr>
<tr>
<td>Logistic</td>
<td>$\mu=5.06713$</td>
<td>0.58030753</td>
<td>0.048046272</td>
<td>0.037381857</td>
<td>51.32986</td>
</tr>
<tr>
<td></td>
<td>$\sigma=0.839694$</td>
<td>0.966736532</td>
<td>0.054193238</td>
<td>0.057461754</td>
<td>19.37559</td>
</tr>
<tr>
<td>Gamma</td>
<td>$a=12.1325$</td>
<td>0.714844377</td>
<td>0.039731558</td>
<td>0.055125167</td>
<td>48.72092</td>
</tr>
<tr>
<td></td>
<td>$b=0.427228$</td>
<td>0.975411891</td>
<td>0.061807918</td>
<td>0.086107126</td>
<td>29.09712</td>
</tr>
<tr>
<td>Normal</td>
<td>$\mu=5.18333$</td>
<td>0.531968115</td>
<td>0.048046272</td>
<td>0.068357386</td>
<td>51.32986</td>
</tr>
<tr>
<td></td>
<td>$\sigma=1.51257$</td>
<td>0.958891503</td>
<td>0.054193238</td>
<td>0.084750925</td>
<td>19.37559</td>
</tr>
</tbody>
</table>

Source: Authors (2020).

#### Adults

<table>
<thead>
<tr>
<th>Models</th>
<th>Estimates</th>
<th>$R^2$</th>
<th>MAE</th>
<th>RMSE</th>
<th>MAPE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Weibull</td>
<td>$a=5.3847$</td>
<td>0.650033856</td>
<td>0.060670537</td>
<td>0.069891164</td>
<td>51.35100561</td>
</tr>
<tr>
<td></td>
<td>$b=4.81248$</td>
<td>0.970243009</td>
<td>0.083152694</td>
<td>0.106086726</td>
<td>34.17287739</td>
</tr>
<tr>
<td>Logistic</td>
<td>$\mu=4.93266$</td>
<td>0.719799091</td>
<td>0.051104191</td>
<td>0.039860805</td>
<td>35.69588254</td>
</tr>
<tr>
<td></td>
<td>$\sigma=0.665318$</td>
<td>0.974237039</td>
<td>0.081689002</td>
<td>0.062409484</td>
<td>25.6754589</td>
</tr>
<tr>
<td>Gamma</td>
<td>$a=16.7809$</td>
<td>0.867704992</td>
<td>0.03945562</td>
<td>0.043527192</td>
<td>29.35098395</td>
</tr>
<tr>
<td></td>
<td>$b=0.29398$</td>
<td>0.987748319</td>
<td>0.080716641</td>
<td>0.091504676</td>
<td>26.84538084</td>
</tr>
</tbody>
</table>
Table 3 shows the results of suitability tests used. Based on the test, the Gamma function is clearly the best distribution, followed by Logistics that best fit the hospital admission data.

To reduce uncertainties in hospital admission estimates, this study compared four candidate distributions (Weibull, Normal, Gamma, and Logistics) in order to select the pdf that best matches hospital admissions data. For this, the monthly data of hospital admissions from 2011 to 2015 in the city of Campo Grande, Brazil, are adjusted by the distributions considered. To determine the effectiveness of the statistical models, the performance of four fit quality tests (coefficient of determination, mean square error, mean absolute error are performed.

Figure 2 shows respectively the histogram of the average hospital admissions of hospitalizations for the years 2011 to 2015 adjusted by the four probability density functions studied and their cumulative frequency adjusted by the four cumulative distribution functions.
Figure 2. Graphs cdf (left) and pdf (right) of the distributions obtained for the monthly averages of hospital admissions for the years (2011-2015).

Source: Authors.

4. Conclusion

The analysis of the monthly values indicates that Gamma distribution fit the data better compared to other distributions considered. This is indicated by the values RMSE. From the values of MAE, Gama also shows better adjustments. It can be concluded that Gamma distribution is suitable for modeling hospital admission data based on the data considered in Campo Grande. This distribution can therefore be used as an alternative distribution that adequately describes hospital admission data. It is suggested that further studies be carried out in other municipalities in order to confirm the use and the importance of this model to assist in epidemiological statistics in public health.

References


Percentage of contribution of each author in the manuscript

Amaury de Souza – 20%
Debora Aparecida da Silva Santos – 20%
José Francisco de Oliveira-Júnior – 20%
Ana Paula Garcia Oliveira – 20%
Elania Barros da Silva – 20%